Reduced MIP Formulation for Transmission Topology Control

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***Abstract*— The standard optimal power flow minimizes generation costs subject to a fixed transmission network topology. Although the co-optimization of network topology and generation resources results in significant congestion cost avoidance, it requires the solution of a mixed integer program (MIP), which is in general intractable for even moderate size systems. The current MIP formulations use the $B\theta$ power flow model, which does not scale with the number of switchable lines or with the number of monitored/contingent facilities, and as such is not amenable to developing tractable topology control (TC) heuristics. This paper introduces the *shift factor* MIP formulation of the TC problem, where line openings are emulated through the use of flow-cancelling transactions. The shift factor formulation is very compact and its size is a function of the number of pairs of monitored/contingent transmission elements and the number of switchable lines. Simulation results on the IEEE 118-bus test system show the superior computational performance of the shift factor formulation as compared to the $B\theta$ formulation for small to medium switchable sets.**

NOMENCLATURE

Vectors are indicated by lower case bold, matrices by upper case bold, and scalars by lower case italic characters indexed appropriately. Upper limits are indicated by an over-bar, and lower limits by an under-bar. Diagonal matrices are denoted with a tilde, and reduced matrices or vectors with a minus sign as a superscript. Sensitivities are indicated with Greek characters.

Indices

m, n	Nodes.
k, ℓ	Lines.
m_ℓ	Line ℓ *from* node.
n_ℓ	Line ℓ *to* node.
τ	Topology.

Topology-Dependent Parameters and Variables

For topology τ,

$\mathbf{A}_\tau$	Incidence matrix.
$\mathbf{A}_\tau^-$	Reduced incidence matrix.
$\tilde{\mathbf{B}}_\tau$	Branch susceptance matrix.
$\mathbf{B}_\tau$	Nodal susceptance matrix.
$\mathbf{B}_\tau^-$	Reduced nodal susceptance matrix.
$\mathbf{f}_\tau$	Vector of flows on transmission elements.
$\underline{\mathbf{f}}_\tau, \overline{\mathbf{f}}_\tau$	Vectors of transmission limits
$\underline{\tilde{\mathbf{F}}}_\tau, \overline{\tilde{\mathbf{F}}}_\tau$	Diagonal matrices of transmission limits
$\boldsymbol{\theta}_\tau$	Vector of nodal voltage angles.
$\mathbf{v}_\tau$	Vector of flow-cancelling transactions.
$\boldsymbol{\Psi}_\tau$	Shift factor matrix.
$\boldsymbol{\Psi}_\tau^{\mathcal{M}}$	Shift factor matrix associated to monitored lines.
$\boldsymbol{\Psi}_\tau^{\mathcal{S}}$	Shift factor matrix associated to switchable lines.
$\boldsymbol{\Phi}_\tau^{\mathcal{SS}}$	PTDF matrix of switchable lines for transfer between switchable line terminals.
$\boldsymbol{\Phi}_\tau^{\mathcal{MS}}$	PTDF matrix of monitored lines for transfer between switchable line terminals.
$\psi_{\ell\tau}^m$	Element of $\boldsymbol{\Psi}$ for line ℓ, node m.
$\phi_{\ell\tau}^{mn}$	PTDF of line ℓ for a transfer from m to n.

Topology-Independent Parameters and Variables

$\mathbf{1}$	Vector of ones.
$\mathbf{0}$	Vector of zeros.
$\mathbf{z}$	Vector with the state of transmission lines.
$\mathbf{c}$	Vector of nodal generation variable cost.
$\mathbf{p}$	Vector of nodal generation.
$\mathbf{p}^-$	Reduced vector of nodal generation.
$\mathbf{l}$	Vector of nodal loads.
$\mathbf{l}^-$	Reduced vector of nodal loads.
M	Very large number.
G	Number of generators.
T	Number of topologies.
Z	Number of switchable lines.
L	Number of transmission lines.
C	Number of monitored/contingent pairs.

I. INTRODUCTION

POWER flows distribute over an AC network following Kirchoff's laws. As such, flows depend on load profile, generation dispatch and transmission topology, including transmission system characteristics, settings and connectivity status. Currently, few transmission branches[1] have flow control devices. The open/closed state of all other branches is typically considered to be fixed or non-controllable in operations decision making, such as economic dispatch (ED). Transmission topology changes tend to be considered as

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[1]In this paper, a transmission branch refers to a facility connecting two nodes of the network, such as a line or a transformer.

inputs to the decision processes, such as a list of pre-specified contingencies, or as a transmission maintenance schedule, and not as a decision variable. Exceptions exist, however. Rule-based decisions like *operating guides* and *special protection schemes* open or close pre-specified breakers upon the occurrence of contingencies or other pre-specified phenomena [1].

The lack of topology control (TC) application has been in spite of research work done in the area over the last decades. For example, corrective control [2]–[4], security enhancements [5], [6] and loss minimization [7], [8] are among the several applications investigated. More recently, topology control has been introduced with the goal of production cost minimization in conjunction with economic dispatch [9]–[11] and unit commitment (UC) [12]. Potential production cost savings enabled by topology control are very promising, with figures of several percentage points in small test systems, which would translate to several billion dollars in annual savings in the U.S. alone. Production cost minimization is the focus of this paper.

Computational complexity has been one of several barriers to the widespread formal application of TC for production cost minimization. The problem has been formulated as a mixed integer linear program (MIP) using the $B\theta$ formulation of transmission constraints [10]. This formulation has been extremely valuable: it showed that the problem can be formulated as a MIP including security constraints, and it enabled a series of simulation analyses on optimal topology control. While the $B\theta$ formulation of transmission constraints has the advantage of preserving the sparsity in the network equation matrices, it suffers from a very large size and poor scalability characteristics. For example, the problem size does not depend on the number of lines whose connectivity is controlled, or on the most relevant monitored/contingency element pairs. Also, for each contingency in the contingency list, a full transmission model is required. As such, the model size explodes with security constraints: in the optimal power flow (OPF) model with TC of the IEEE 118-bus test system, $n-1$ security constraints require 63,000 variables and 200,000 constraints, compared to approximately 500 variables and 1000 constraints without security constraints [11]. In terms of solution time, the performance is not acceptable: the integrality gap of the security-constrained OPF (SCOPF) with TC was about 60% after six days of run time [11]. While there have been significant improvements in MIP solvers and computer resources since the publication of [11], and while formulations have been improved with the addition of symmetry breaking and anti-islanding constraints [13], [14], the resulting computation times are still very far from the required times for deployment in operations and planning. This is especially the case considering the dimension of practical network models as compared with those of test systems.

To overcome computational tractability issues, heuristic approaches have been developed for the TC problem. Some of these heuristics use the ($B\theta$) MIP formulation [11], but the reduction in computational effort is not sufficient for practical use. Alternative approaches based on sensitivity analysis have been very successful in reducing computational times in an OPF setting, where dispatch is optimized for a single time period [13], [15]–[17]. However, the extension of these tractable approaches to a dynamic setting (e.g., multi-interval ED and UC) is not trivial. In dynamic decision making, dynamic constraints, such as maximum number of breakers that can change state on a given interval and maximum switching frequency, combined with other constraints such as the total number of breakers that can be open at any point in time, require the topology optimization over a much longer horizon than a single time interval.

This paper introduces a MIP formulation of the TC problem for use in both static and dynamic decision making including SCOPF, security-constrained ED (SCED) and UC (SCUC), and longer time frame problems. The formulation is based on the use of shift factors, consistent with the traditional transmission modeling approach used in energy management systems (EMS) and market management systems (MMS). The opening of breakers is emulated by the use of *flow-cancelling transactions*, e.g., pairs of injections and withdrawals at the end of opened lines that make the total flow through the line *interface* with the rest of the system to be zero, rather than by changing the line admittance. This modeling approach is analogous to the use of phase angle regulators to set the flow on the line interface to zero. Compared to the ($B\theta$) MIP formulation, the formulation introduced in this paper is very compact, and its size is a function of the number of pairs of monitored/contingent transmission elements and the number of switchable lines. The formulation is useful to model TC when few constraints need to be explicitly enforced, and also to develop sequential heuristics that include dynamic constraints.

The rest of the paper has six sections. Section II presents the basic power flow modeling and notation. Section III provides an overview of the $B\theta$ formulation of TC. Section IV describes the modeling of line openings using flow-cancelling transactions. Section V introduces the reduced TC formulation using shift factors and flow-cancelling transactions. Section VI compares the computational performance of the two formulations for the SCOPF with TC. Section VII gives concluding remarks and describes future work in the area.

II. Power Flow Model

This section presents basic underlying OPF modeling assumptions for use in the different TC formulations.

Consider a power system in which linearized lossless dc assumptions hold. This system has buses $n = 1, \ldots, N$ and branches $\ell = 1, \ldots, L$. Each line ℓ is associated with an ordered pair of nodes (m_ℓ, n_ℓ), with the convention that the flow direction of line ℓ is $from$ node m_ℓ and to node n_ℓ. Let bus N be the reference bus, which has voltage angle 0; $\tilde{\mathbf{B}}$ be the branch susceptance matrix, a diagonal matrix with the line susceptances as its elements; and $\mathbf{A}$ be the incidence matrix, an $L \times N$ matrix which for each row ℓ has elements -1 and 1 in the columns corresponding to the from and to nodes of line ℓ, respectively, and 0 for all other nodes. The reduced incidence matrix, $\mathbf{A}^-$ is a submatrix of $\mathbf{A}$, without the column corresponding to the reference bus.

At any point in time, a subset of the transmission lines may be disconnected (open), either due to contingencies, or to planned actions including maintenance or as part of topology control decisions. The resulting topology τ of the transmission system is characterized by the incidence matrix $\mathbf{A}_\tau$, which consists of all rows of $\mathbf{A}$ that are associated with the branches connected (closed) in topology τ. The corresponding reduced incidence matrix is denoted by $\mathbf{A}_\tau^-$. The nodal susceptance matrix $\mathbf{B}_\tau$ for topology τ is given by

$$\mathbf{B}_\tau = -\mathbf{A}_\tau' \tilde{\mathbf{B}} \mathbf{A}_\tau \tag{1}$$

and the reduced nodal susceptance matrix $\mathbf{B}_\tau^-$ is given by

$$\mathbf{B}_\tau^- = -\mathbf{A}_\tau^{-\prime} \tilde{\mathbf{B}} \mathbf{A}_\tau^-. \tag{2}$$

For topologies without islands, $\mathbf{B}_\tau^-$ is invertible, while $\mathbf{B}_\tau$ is not invertible for any topology.

The nodal power balance equations, which state that the net load at each bus equals the net line flow to the bus, can be expressed in terms of $\mathbf{A}_\tau$ and the vector $\mathbf{f}_\tau$ of power flows on each transmission line, as (3),

$$(\mathbf{l} - \mathbf{p}) = \mathbf{A}_\tau' \mathbf{f}_\tau, \tag{3}$$

where $\mathbf{p}$ and $\mathbf{l}$ are the vectors of nodal power generation and loads, respectively. Let $\mathbf{p}^-$ and $\mathbf{l}^-$ be the reduced vectors of nodal power generation and loads. The power balance equation for all buses except the reference bus are given by

$$(\mathbf{l}^- - \mathbf{p}^-) = \mathbf{A}_\tau^{-\prime} \mathbf{f}_\tau. \tag{4}$$

Note that generation and load in this paper are assumed to be independent of the topology, although they need not be (e.g., under corrective control). The line flow vector $\mathbf{f}_\tau$ is given by (5),

$$\mathbf{f}_\tau = \tilde{\mathbf{B}} \mathbf{A}_\tau \boldsymbol{\theta}_\tau = \tilde{\mathbf{B}} \mathbf{A}_\tau^- \boldsymbol{\theta}_\tau^- \tag{5}$$

where $\boldsymbol{\theta}_\tau$ is the vector of nodal voltage angles, and $\boldsymbol{\theta}_\tau^-$ is the reduced angle vector, without the 0 entry corresponding to the reference bus. From (3)-(5), the nodal power equations are obtained in (6),

$$(\mathbf{p} - \mathbf{l}) = \mathbf{B}_\tau \boldsymbol{\theta}_\tau \tag{6}$$

and the reduced nodal power equations are given by (7),

$$\left(\mathbf{p}^- - \mathbf{l}^-\right) = \mathbf{B}_\tau^- \boldsymbol{\theta}_\tau^- \tag{7}$$

From (5) and (6), the power flows can be expressed as an explicit function of the loads and generation,

$$\begin{aligned} \mathbf{f}_\tau &= \tilde{\mathbf{B}} \left[\mathbf{A}_\tau^- \left(\mathbf{B}_\tau^-\right)^{-1}, \mathbf{0}\right] (\mathbf{p} - \mathbf{l}) && (8) \\ &= \boldsymbol{\Psi}_\tau (\mathbf{p} - \mathbf{l}). && (9) \end{aligned}$$

The *transmission sensitivity matrix* $\boldsymbol{\Psi}_\tau$ [18], also known as the *injection shift factor matrix*, gives the variations in flows for each line in topology τ due to changes in the nodal injections, with the reference bus assumed to ensure the real power balance. The shift factor matrix is a function of the transmission element susceptances and the topology (τ). The shift factor for line ℓ and node n is denoted by $\psi_{\ell\tau}^n$.

The *power transfer distribution factor* $\phi_{\ell\tau}^{mn}$, or *PTDF*, gives the sensitivity of the flow on line ℓ with respect to a power transfer from node m to node n under topology τ, and can be expressed in terms of shift factors as [19]

$$\phi_{\ell\tau}^{mn} = \psi_{\ell\tau}^m - \psi_{\ell\tau}^n. \tag{10}$$

III. Bθ Topology Control Formulation

The typical MIP formulations of topology control problems model transmission flows using (5), i.e., explicitly keeping the susceptances as inputs and voltage angles as decision variables [10]–[12], [20], hence the name $B\theta$ formulation. The supply-demand balance is enforced at the nodal level using (3). The reason for this model choice is that the linear inclusion of binary variables associated with the inclusion or removal of branches is more apparent and intuitive. In contrast, shift factor power flow models have a nonlinear dependence on susceptances and connectivity (equation (9)).

For notational simplicity, and without loss of generality, assume there is at most one generator at each bus, which has a constant marginal cost of generation. The SCOPF with TC minimizes generator costs to serve load (11) subject to physical constraints such as generator (12) and line (13) limits ($\underline{\tilde{\mathbf{F}}}_\tau$ and $\overline{\tilde{\mathbf{F}}}_\tau$ are diagonal matrices with the lower and upper line limits are their elements, respectively). The incorporation of TC requires the addition of a binary variable (17), which renders the problem a MIP. This variable indicates the line status, i.e., takes the value of 1 if the line is closed, and 0 if open. The power balance at each node is enforced by (14). In addition, (15) and (16) define flows as a function of voltage angles, where M is a sufficiently large number and the first two terms are from (5). Note that this formulation computes angles for all nodes and flows on all lines for for each contingency topology τ of a pre-specified contingency list.

$$\mathcal{C} = \min_{\mathbf{p},\boldsymbol{\theta},\mathbf{f},\mathbf{z}} \mathbf{c}'\mathbf{p} \tag{11}$$

$$\begin{aligned} \text{subject to} \quad & \underline{\mathbf{p}} \le \mathbf{p} \le \overline{\mathbf{p}}, && (12) \\ & \underline{\tilde{\mathbf{F}}}_\tau \mathbf{z} \le \mathbf{f}_\tau \le \overline{\tilde{\mathbf{F}}}_\tau \mathbf{z}, \quad \forall \tau && (13) \\ & \mathbf{A}_\tau' \mathbf{f}_\tau + \mathbf{p} - \mathbf{l} = 0, \quad \forall \tau && (14) \\ & \tilde{\mathbf{B}} \mathbf{A}_\tau \boldsymbol{\theta}_\tau - \mathbf{f}_\tau + (\mathbf{1} - \mathbf{z}) M \ge 0, \quad \forall \tau && (15) \\ & \tilde{\mathbf{B}} \mathbf{A}_\tau \boldsymbol{\theta}_\tau - \mathbf{f}_\tau + (\mathbf{1} - \mathbf{z}) M \le 0, \quad \forall \tau && (16) \\ & z_\ell \in \{0, 1\}, \quad \forall \ell && (17) \end{aligned}$$

In the remainder of the paper, problem (11)-(17) is referred to as the $B\theta$ TC formulation. Let the number of generators be G, the number of contingency topologies be T and the number of switchable lines be Z. The $B\theta$ TC formulation has approximately $G + (N-1)T + LT + Z$ decision variables (the approximation stems from the fact that contingent topologies will usually have less than L lines connected), and $2G + 4LT + NT + Z$ constraints. As such, the problem dimension is essentially insensitive to the number of switchable lines and monitored transmission constraints.

The following sections introduce a formulation whose size is a function of both the number of switchable lines and the number of monitored transmission constraints.

IV. Flow-Cancelling Transactions

There are two approaches for modeling a branch outage. The direct approach is to remove the line from the susceptance matrix, or make the line susceptance zero, as is done in the $B\theta$ formulation detailed in the previous section. An alternative approach is to maintain the original topology and susceptances, but to apply a power transfer that would cancel the flow on the interfaces between the rest of the system and the line of interest, so that from the point of view of the rest of the system, the line is outaged.

The modeling approach of representing outages as a flow-cancelling transaction is widely known, for example as a tool to derive line outage distribution factors [19]. Consider first the derivation of a flow-cancelling transaction for a single line. To model the outage of line k, let m'_k and n'_k be infinitely close to the terminal nodes m_k and n_k along line k (Fig. 1). Let there be a transaction from m'_k to n'_k whose magnitude $v_{k\tau}$ is such that the impact of the trasaction on the rest of the system is equivalent to the opening of line k. To meet this condition, the flow-cancelling transaction must make the flow on the interfaces between the rest of the system and line k, i.e., each of the infinitesimaly short lines m'_k to m_k and n'_k to n_k, to be zero. Using the PTDF definition,

$$f_{k\tau} - \left(1 - \phi_{k\tau}^{m'_k n'_k}\right) v_{k\tau} = 0. \tag{18}$$

Hence,

$$v_{k\tau} = \frac{f_{k\tau}}{1 - \phi_{k\tau}^{m'_k n'_k}}. \tag{19}$$

In general, the vector of flow-cancelling transactions that model the outage of a set $\mathcal{S}$ of lines, which can be easily obtained by applying the principle of superposition, meets the condition (18) for all lines in the set [21],

$$\mathbf{f}_\tau^{\mathcal{S}} - \left(\mathbf{I} - \boldsymbol{\Phi}_\tau^{\mathcal{S}\mathcal{S}}\right) \mathbf{v}_\tau^{\mathcal{S}} = \mathbf{0}. \tag{20}$$

In here, the superscript $\mathcal{S}$ indicates the vectors of variables associated to set $\mathcal{S}$, and $\boldsymbol{\Phi}_\tau^{\mathcal{S}\mathcal{S}}$ is the matrix of PTDFs for transactions between the terminal points of lines in $\mathcal{S}$, with respect to the flows of lines in $\mathcal{S}$. We term such matrix the *self-PTDF matrix* of set $\mathcal{S}$.

Next section discusses the application of flow-cancelling transactions to a MIP topology control formulation.

V. Reduced Topology Control Formulation

Usually, for a given topology τ there are only a few transmission elements that could limit transfers in practice, and therefore are monitored. For example, if lines k and ℓ are parallel, it may be the case that if all transmission constraints are met in the base topology, ensuring that line ℓ is not overloaded after the outage of parallel line k may be sufficient to ensure that no transmission constraint violation occurs with the outage of line k. In other words, all other contingency constraints in (13) would not bind. Even in the base case, with no contingencies, the number of limiting transmission facilities in actual systems is only a small fraction of the total number of lines and transformers.

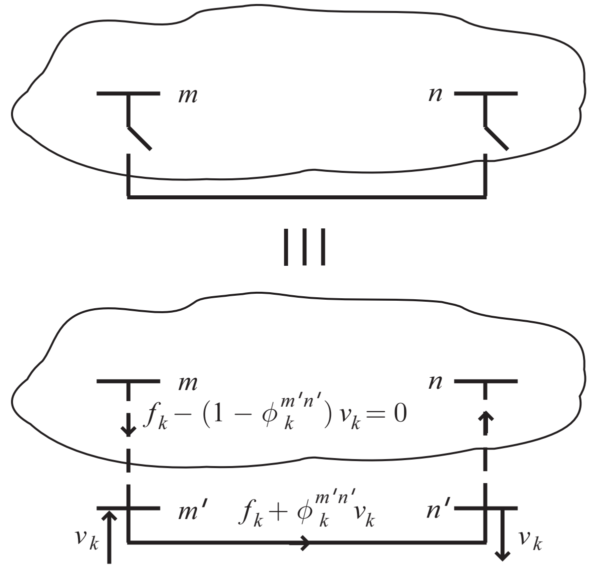

Fig. 1. Opening line k (top) is equivalent from the point of view of the rest of the system as inserting a flow-cancelling transaction at virtual buses m' and n', infinitely close to m and n, respectively, and along line k (bottom).

To reduce the problem size and only model constraints that are significant, the SCOPF problem is usually formulated using (9) instead of (5). This allows the elimination of the nodal equations (6) and of all equations in (9) that are not related to monitored facilities. While the resulting formulation is significantly reduced, both in terms of constraints as well as variables (e.g., $\boldsymbol{\theta}$ is no longer modeled explicitly), the remaining equations are significantly more dense, since the shift factor matrix $\boldsymbol{\Psi}_\tau$ is a dense matrix while $\mathbf{B}_\tau$ is very sparse. Still, due to the very small number of monitored constraints, the dense formulation solves faster in practice than the sparse formulation.

Let the superscript $\mathcal{S}$ denote variables or parameters related to facilities in the switchable set. Let $\mathbf{v}_\tau$ be the vector of flow-cancelling transactions to model the state of all switchable lines under contingent topology τ, $\underline{\tilde{\mathbf{F}}}_\tau^{\mathcal{S}}$ and $\overline{\tilde{\mathbf{F}}}_\tau^{\mathcal{S}}$ be diagonal matrices with the transmission limits of switchable facilities under topology τ, $\boldsymbol{\Psi}_\tau^{\mathcal{S}}$ be the reduced shift factor matrix associated to switchable lines under topology τ, and $\boldsymbol{\Phi}_\tau^{\mathcal{S}\mathcal{S}}$ be the self-PTDF matrix of the switchable set, under topology τ. For the open switchable lines, equation (20) needs to be enforced, while for closed lines, (9) needs to be enforced.

This is achieved by the following set of constraints,

$$\underline{\tilde{\mathbf{F}}}_\tau^{\mathcal{S}}\mathbf{z} \leq \mathbf{\Psi}_\tau^{\mathcal{S}}\left(\mathbf{p}-\mathbf{l}\right) + \left(\mathbf{\Phi}_\tau^{\mathcal{SS}} - I\right)\mathbf{v}_\tau \leq \overline{\tilde{\mathbf{F}}}_\tau^{\mathcal{S}}\mathbf{z}, \quad \forall\tau \tag{21}$$

$$-M\left(\mathbf{1}-\mathbf{z}\right) \leq \mathbf{v}_\tau \leq M\left(\mathbf{1}-\mathbf{z}\right), \quad \forall\tau. \tag{22}$$

where $\mathbf{z}$ indicates the state of the branches, as in Section III. Constraints (22) force the flow-cancelling transactions to be 0 for all closed lines, while holds them not restricted for all open lines, as M is a sufficiently large number.

Let variables and parameters related to monitored facilities be denoted with superscript $\mathcal{M}$. In the remainder of the paper, a monitored facility means a monitored facility that is not switchable. Let $\underline{\mathbf{f}}_\tau^{\mathcal{M}}$ and $\overline{\mathbf{f}}_\tau^{\mathcal{M}}$ be vectors with the transmission limits of monitored, non-switchable facilities under topology τ, $\mathbf{\Psi}_\tau^{\mathcal{M}}$ be the reduced shift factor matrix associated with monitored lines under topology τ, and $\mathbf{\Phi}_\tau^{\mathcal{MS}}$ be the PTDF matrix of transactions between the terminal nodes of each switchable line with respect to lines in the monitored set, under topology τ. For monitored facilities, the flow constraints incorporate the impacts of flow-cancelling transactions for switchable lines, and is given by

$$\underline{\mathbf{f}}_\tau^{\mathcal{M}} \leq \mathbf{\Psi}_\tau^{\mathcal{M}}\left(\mathbf{p}-\mathbf{l}\right) + \mathbf{\Phi}_\tau^{\mathcal{MS}}\mathbf{v}_\tau \leq \overline{\mathbf{f}}_\tau^{\mathcal{M}}, \quad \forall\tau. \tag{23}$$

The resulting formulation of the SCOPF with TC is

$$\mathcal{C} = \min_{\mathbf{p},\mathbf{v},\mathbf{z}} \mathbf{c}'\mathbf{p} \tag{24}$$

$$\text{subject to} \tag{25}$$

$$\mathbf{1}'\left(\mathbf{p}-\mathbf{l}\right) = 0, \tag{26}$$

$$\underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}}, \tag{27}$$

$$\underline{\mathbf{f}}_\tau^{\mathcal{M}} \leq \mathbf{\Psi}_\tau^{\mathcal{M}}\left(\mathbf{p}-\mathbf{l}\right) + \mathbf{\Phi}_\tau^{\mathcal{MS}}\mathbf{v}_\tau \leq \overline{\mathbf{f}}_\tau^{\mathcal{M}}, \quad \forall\tau \tag{28}$$

$$\underline{\tilde{\mathbf{F}}}_\tau^{\mathcal{S}}\mathbf{z} \leq \mathbf{\Psi}_\tau^{\mathcal{S}}\left(\mathbf{p}-\mathbf{l}\right) + \left(\mathbf{\Phi}_\tau^{\mathcal{SS}} - I\right)\mathbf{v}_\tau \leq \overline{\tilde{\mathbf{F}}}_\tau^{\mathcal{S}}\mathbf{z}, \quad \forall\tau \tag{29}$$

$$-M\left(\mathbf{1}-\mathbf{z}\right) \leq \mathbf{v}_\tau \leq M\left(\mathbf{1}-\mathbf{z}\right), \quad \forall\tau \tag{30}$$

$$z_\ell \in \{0,1\}, \quad \forall\ell \tag{31}$$

Problem (24)-(31), referred to as the shift factor TC formulation, is equivalent to the $B\theta$ formulation in the sense that both would yield the same optimal solution. However, the problem size and characteristics are quite different. The shift factor TC formulation has $G+TZ+Z$ decision variables and $1+2G+2C+4TZ+Z$ constraints, where C is the number of monitored/contingency pairs. If the number of switchable lines and monitored/contingent facility pairs are relatively small, the shift factor formulation is significantly smaller than the $B\theta$ formulation.

VI. Simulation Results

We tested the shift factor TC formulation on the IEEE 118-bus test system. The version of the test system employed [22] consists of 118 buses, 54 generators, and 194 branches, all of which are connected. The load is 3,668 MW. The generation economic model developed for this test system is detailed in [15]. To study the formulations' performances under different system conditions, we maintain a fixed load and perform a Monte Carlo simulation where the fuel costs and the available wind generation are randomly varied, with 100 samples taken. The details of the Monte Carlo simulation are included in [13].

We implemented the $B\theta$ and the shift factor TC formulations in AIMMS 3.12, and used CPLEX 12.4 to solve the resulting MIPs. The TC formulations implemented and tested include a few constraints not detailed in the previous section for simplicity. These are a set of connectivity constraints that ensure that each generator and load bus is connected by at least two lines, and a set of symmetry-breaking constraints that provide a preferred ordering for each group of identical parallel lines.

The transmission constraints enforced in both TC formulations were obtained by running the price difference and line profit TC heuristics detailed in [15] with full n-1 security-constraint enforcement, and for 100 samples of available wind generation and fuel costs. If a given constraint was binding in at least one heuristic iteration for at least one of the 100 samples, the constraint was included in the list for enforcement. A constraint is characterized by a pair of a monitored facility and a contingent facility. A total of 142 such constraints were found, including 63 different contingencies.

The cardinality of the switchable set has a very significant impact on the shift factor formulation size. As such, we limited and varied the number of lines included in the switchable set using a priority list. The priorities were assigned using the number of samples in which each line was disconnected by the price difference and line profit TC heuristics discussed in the previous paragraph. The lines more frequently disconnected by the heuristics were given higher priority. This procedure seeks to emulate an operational environment, where candidates that performed well in the past are considered first. The priority list is given in the Appendix.

Figure 2 shows the average solution time over the 100 scenarios for both formulations, where the convergence criterion is an optimality gap of up to 0.001. As a reference, the savings with 20 lines in the switchable set amount to over 4% of the total production costs in the case with full topology. Note that the shift factor formulation results in lower average computational time for all switchable set sizes. For very small switchable sets, the computational effort is about one order of magnitude lower for the shift factor formulation as compared to the $B\theta$ formulation. The computation time savings are reduced as the switchable set increases. For a switchable set of 20 lines, the shift factor formulation solves in 2/3 of the time that the $B\theta$ formulation takes. Simulations were run on a workstation with two 2.93 GHz Intel Xeon(R) processors and 24 GB of RAM.

VII. Concluding Remarks

We have developed a MIP-based TC formulation that uses shift factors and that is consistent with the SCED and SCUC formulations currently used in practice. In contrast with the published $B\theta$ TC formulation, the shift factor TC formulation

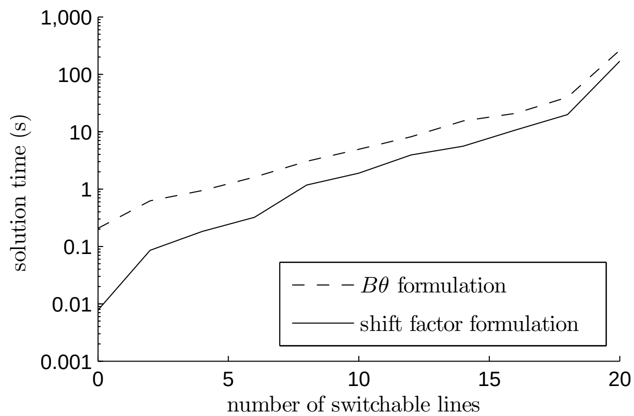

Fig. 2. Average solution time as a function of the number of switchable lines.

is compact and very scalable, and its size depends heavily on the number of decision variables (switchable lines) and transmission constraints (monitored lines and contingencies). While the shift factor formulation is significantly denser than the $B\theta$ formulation, it solves faster, especially for TC problems with a reduced number of switchable lines and a reduced number of pre-specified monitored/contingent transmission facility pairs.

Several assumptions taken in this paper can be easily relaxed. While the modeling was done with lossless DC power flow assumptions for simplicity in the explanation, the same methodology can be applied with any linearized power flow assumptions. For example, the linearization gap, or bias, can easily be included in the formulation. Also, the formulation can be used in multi-period SCED and SCUC. In these problems, constraints on the maximum frequency of switching, anti-islanding constraints, maximum number of switchings constraints can be enforced. Finally the cost of switching can be added in the objective function should switching costs be important in practice.

Future work includes the development of iterative heuristics using this formulation, the addition of losses, and the study of dynamics in ED and UC formulations.

APPENDIX

PRIORITY OF SWITCHABLE LINES FOR IEEE 118-BUS TEST SYSTEM

The priority list in Table I was used to construct switchable sets. As such, a switchable set with Z lines consists of the Z lines with highest priority. The criteria to build the priority list are described in Section VI.

TABLE I
SWITCHING PRIORITY

Priority	From	To	Ckt	Priority	From	To	Ckt
1	31	32	1	16	69	77	1
2	49	66	1	17	59	61	1
3	27	32	1	18	37	40	1
4	3	12	1	19	59	60	1
5	61	62	1	20	64	65	1
6	49	66	2	21	15	19	1
7	69	70	1	22	65	66	1
8	46	47	1	23	80	96	1
9	77	82	1	24	64	61	1
10	55	59	1	25	30	38	1
11	24	70	1	26	70	71	1
12	54	59	1	27	92	100	1
13	56	59	1	28	11	12	1
14	56	59	2	29	94	100	1
15	56	59	3	30	23	25	1

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